12.57. Solve: **(a)** Taking the logarithm of both sides of $v^p = Cu^q$ gives

$$[\log(v^p) = p\log v] = [\log(Cu^q) = \log C + q\log u] \Rightarrow \log v = \frac{q}{p}\log u + \frac{\log C}{p}$$

But $x = \log u$ and $y = \log v$, so x and y are related by

$$y = \left(\frac{q}{p}\right)x + \frac{\log C}{p}$$

(b) The previous result shows there is a linear relationship between x and y, hence there is a linear relationship between log u and log v. The graph of a linear relationship is a straight line, so the graph of log v-versus-log u will be a straight line.

(c) The slope of the straight line represented by the equation $y = (q/p)x + \log C/p$ is q/p. Thus, the slope of the log v-versus-log u graph will be q/p.

(d) From Newton's theory, the period T and radius r of an orbit around the sun are related by

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

This equation is of the form $T^p = Cr^q$, with $p = 2$, $q = 3$, and $C = 4\pi^2/GM$. If the theory is correct, we *expect* a graph of log T-versus-log r to be a straight line with slope $q/p = 3/2 = 1.500$. The *experimental measurements* of actual planets yield a straight line graph whose slope is 1.500 to four significant figures. Note that the graph has nothing to do with theory—it is simply a graph of measured values. But the fact that the shape and slope of the graph agree precisely with the prediction of Newton's theory is strong evidence for its correctness.

(e) The predicted y-intercept of the graph is log C/p, and the experimentally determined value is 9.264. Equating these, we can solve for M. Because the planets all orbit the sun, the mass we are finding is $M = M_{\text{sun}}$.

$$\frac{1}{2}\log C = \frac{1}{2}\log\left(\frac{4\pi^2}{GM_{\text{sun}}}\right) = -9.264 \Rightarrow \frac{4\pi^2}{GM_{\text{sun}}} = 10^{-18.528} = \frac{1}{10^{18.528}}$$

$$\Rightarrow M_{\text{sun}} = \frac{4\pi^2}{G}\cdot 10^{18.528} = 1.996\times 10^{30}\ \text{kg}$$

The tabulated value, to three significant figures, is $M_{\text{sun}} = 1.99 \times 10^{30}$ kg. We have used the orbits of the planets to "weigh the sun!"